The Influence of Loads Superposition, Deterioration Due to Cracks and Residual Stresses on the Strength of Tubular Junction

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In the paper the interaction of several loads like pressure, axial force, bending moment and torsional moment are analyzed, taking into account the deterioration due to cracks and the influence of residual stresses. A nonlinear, power law, of structure material is considered. General relationships for total participation of specific energies introduced in the structure by the loads, as well as for the critical participation have been proposed. On these bases: - a new strength calculation methods was developed; – strength of tubular cracked structures and of cracked tubular junction subjected to combined loading and strength were analyzed. Relationships for critical state have been proposed, based on dimensionless variables. These theoretical results fit with experimental date reported in literature. On the other side stress concentration coefficients were defined. Our one experiments onto a model of a pipe with two opposite nozzles have been achieved. Near one of the nozzles is a crack on the run pipe. Trough the experiments the state of stress have been obtained near the tubular junction, near the tip of the crack and far from the stress concentration points. On this basis the stress concentration coefficients were calculated.

Keywords: strength of tubular structure; crack; residual stress; tubular junction strength; critical parameters for cracked structures

The shell intersections are widely used in process industries (chemical, petrochemical) pipeline transportation, various machines, industrial constructions etc. The branch junctions and pressure vessels – nozzles connections are such example. It is necessary to consider the stress state in the intersection of the shells under mixed-mode or combined loading; generally here occur significant plastic deformations.

Skopinsky et al. [1] underlines: in practice [2 - 4], only two methods are the most commonly used in the analysis of pressure vessels with nozzles and piping tees: the Twice Elastic Slope method and the Tangent Intersection method. Both methods are based on a characteristics load – deformation curve and are empirical procedure.

A plastic work criteria proposed in the paper [5] is based on the plastic work dissipated in structure of ductile material. The three methods have drawbacks, which consist of the subjective choice of the deformations parameters, or in definition the plastic limit load.

Other methods for inelastic analysis of shell intersections under combined loading were expound by Skopinsky et al. [1]. It is, to underline the experimental investigations and/or the results of tests and finite element analysis of branch junctions [1, 6 - 8] under different combined loadings, which show that the problems of shell intersections subjected to combined and/or mixed – mode load has not been investigated sufficiently.

In the nonlinear analyses, generally, the authors have represented the interdependence between the loads ratio, such as

$$\overline{p} = p/p_{cr}\ ;\quad \overline{M}_b = M_b/M_{b,cr}\ ;$$

$$\overline{F} = F/F_{cr}\ ;\quad \overline{M}_t = M_t/M_{t,cr}\ \text{ etc...},$$

where the denominators are the critical values of the numerator (p – pressure; M_b – bending moment; F – axial force; M_t – torsional moment). Often the critical values were considered as plastic limits (p_y; M_{by}; F_y; Mt_{ty} etc.) corresponding to yield stress (σ_y).

Logical the curve in such a diagram must contain the points of values 1.0 on each axis; for example in the $\overline{p}-\overline{M}$ interdependence represented in figure 1. This is not the case in some papers.

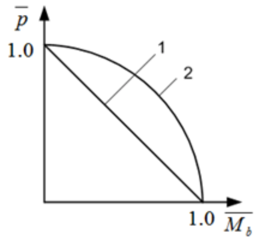

Fig. 1. The interdependence between the pressure ratio, $\overline{p}$, and bending moment ratio, $\overline{M}_b$ (qualitative): 1 - linear correlation; 2 – nonlinear correlation

This paper consists in a new approach of shells junctions under different combined loadings, starting with the principle of critical energy and using the nonlinear, power law, behavior of the structural components. The influence of deterioration as well as of the residual stresses are introduced.

State of loading in the intersection of shells

In the both shells represented in figure 2, in the junction area, the stress often exceed the yield stress. The area near the intersection, influenced by the combined loadings (fig. 2) is extend on the distance l_2 for the nozzle and l_1 and l_3, respectively, for the vessel.

At stresses below the yield stress the superposition of stresses due to different loads may be summing algebraic because the material behavior is linear - elastic. If the stresses exceed the yield stress one can not use the algebraic summation because in this case the material behavior is nonlinear.

In this case the principle of critical energy (PCE) must be used [9-12]. In correlations with this principle one uses critical values of forces, bending moments, stresses etc.

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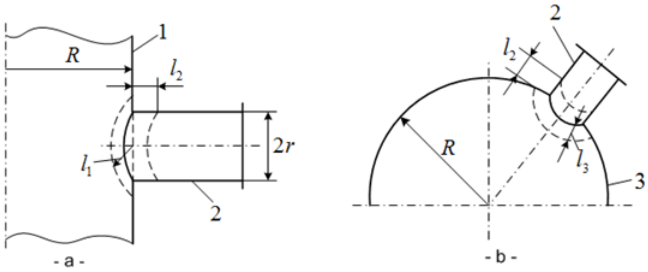

Fig. 2. Shells intersections: *a* – two cylindrical shells (piping branch junction or pressure vessel, *1*, with a nozzle *2*); *b* – spherical vessel (*3*) with radial nozzle, *2*.

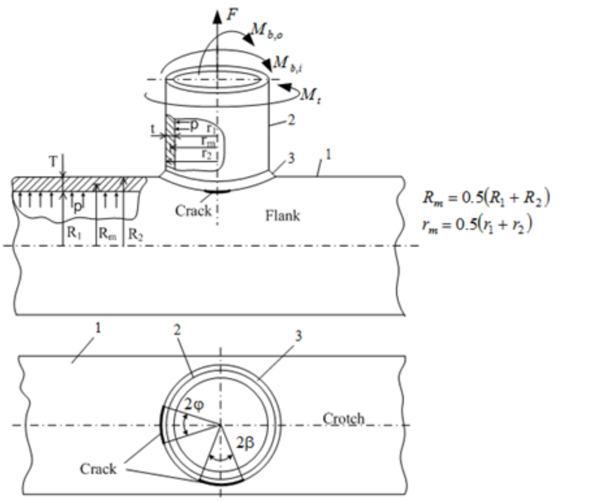

Fig. 3. Tubular junction loaded by internal pressure, *p*, axial force, *F*, out-of plane bending moment, $M_{b,0}$ in-plane bending moment $M_{b,i}$ and torsional moment; M_t. 1 – run pipe; 2 – branch pipe (nozzle); 3 – weld

In general, the critical stress, as well as the critical parameter (pressure, axial force, bending moment etc.) is that value of stress or of parameter that determines the taking out of use or the destruction of a sample or a mechanical structure. The value of critical stress or parameter can be chosen; for example the value which determines the yielding of material (corresponding to yield stress), or the value which determines the failure (corresponding to ultimate stress). The critical stress or critical parameter is marked by subscript cr. For example σ_{cr} is the critical stress, F_{cr} is the critical axial force etc.

Until recently, strength calculation of structures static loaded referred to materials with linear - elastic behavior. Components and structures are usually calculated and constructed according to stress principles: the equivalent stress must be less or equal to an allowable value.

A first way consists in resort to the principle of critical energy [9-12] which introduces the concept of specific energy participation (the energy of a unit volume, J/m^3 or of a unit mass J/kg).

In the case of fatigue loading it was found experimentally that when specimens without cracks are under load, fatigue strength begins to decrease after a number of cycles $N > N_f$, while at specimens with cracks, fatigue strength begins decrease if $N > N^*_f$, where $N^*_f < N_f$. One can see that when $N_f \leq N^*_f$, the critical crack length a_{cr} is constant, while when $N > N^*_f$, the crack will increase continuously with an increasing number of load cycles [13; 14].

Many mechanical structures undergo defects or cracks, as well as welding residual stresses.

The problem is how to calculate the mechanical structures taking into account: – the influence of cracks and residual stresses; – the nonlinear behavior when stress is higher the yield stress.

On both side of the weld there are residual stresses. The design of such shells junctions must take into account the deterioration due to cracks, as well as the influence of residual stresses. These, together with the nonlinear behaviour beyond the yield stress, will be considered in this paper.

Critical stresses and critical loads

For generality one choices a nonlinear, power law, behavior of structure material, under normal stress, σ, and shear stress, τ,

$$\left.\begin{aligned}\sigma &= M_\sigma \cdot \varepsilon^k;\\ \tau &= M_\tau \cdot \gamma^{k_1},\end{aligned}\right\} \qquad (1)$$

where ε is the strain; γ is the shear strain; M_σ, M_τ, k and k_1 are material constants. In our recent papers [15 - 17], on the basis of *principle of critical energy*, there have been proposed the following relations for the critical stresses of tubular specimens with cracks,

$$\left.\begin{aligned}\sigma_{cr}(a;c) &= \sigma_{cr}\cdot\left[1-D(a_\sigma;c)\right]^{\frac{1}{\alpha+1}};\\ \tau_{cr}(a;c) &= \tau_{cr}\cdot\left[1-D(a_\tau;c)\right]^{\frac{1}{\alpha_1+1}},\end{aligned}\right\}, \qquad (2)$$

where the total deterioration $D(a_\sigma; c)$ depends on the crack depth $a \equiv a_\sigma$ and the crack length 2c in the direction perpendicular to the direction of the normal stress σ, while deterioration $D(a_\tau; c)$ depends on the depth of the crack $a \equiv a_\tau$ and the crack length $2c$ in the direction of shear stress τ.

The relations proposed in our papers for yield loading in tubular cylindrical specimens with cracks, generally can be written as in eq. (2), namely

$$Y_L = Y_y \cdot \left|1-\left(D(a;c)\right)^{0.5}\right|, \qquad (3)$$

where Y_L is the limit load of the cracked tubular specimen; Y_y is the limit load of the crackless tubular specimen and α + 1 were replaced by 2, because the behavior at $Y \leq Y_y$ is linear.

Based on this relationship, yield loads in tubular specimens with cracks (F_L; $M_{b,L}$; p_L) were written as a product of the yield load of the specimen without cracks (F_y; $M_{b,y}$; p_y) and a bracket comprising damage D(a;c) [13; 18].

To see the effect of crack location, for the external cracks, one uses the outer radius, R_2, instead of mean radius, R_m, when calculate the limit loadings, F_y, M_y and p_y. For thin walled cylinders the results are approximately the same. The effect of crack location is not significant for shorter cracks [19].

On the basis of Morozov's criterion of rupture for cracked sample [20; 21] one may write,

$$\sigma_{cr}(a;c) = \sigma_{cr}\cdot\left[1-D(c)\right]^{0.5}, \qquad (4)$$

where $D(c)=c/c_{cr}$ with 2*c* the length of the crack and c_{cr} its critical value.

By comparing with eq. (2) in this case α=1, wich corresponds to linear elastic behaviour.

Similar, on the basis of Andreikiv criterion [22], the following failure criterion may be writen,

$$\varepsilon_{cr}(c) = \varepsilon_{cr}\cdot\left[1-D(c)\right]^{1/m}, \qquad (5)$$

where $\varepsilon_{cr}(c)$ and ε_{cr} are the effective and the critical strain, while *m* is an empirical exponent.

The empirical relations (4) and (5) are similar to theoretical relations (2) and (3), but takes into account only the length of the crack.

Strength calculation based on allowable stresses for a structure with a crack

a. *Classical strength calculation methods* currently used in official calculation norms are deterministic methods based on the condition,

$$\sigma_{eq} \le \sigma_{al}, \qquad (6)$$

where σ_{eq} is the equivalent stress, calculated by using a strength theory (Tresca or von Mises theory). The allowable stress in this case $\sigma_{al}=\sigma_{cr}/\sigma_{\sigma}$, where $c_{\sigma}>1$ is the safety coefficient.

This method of strength calculation can be applied to structures with cracks, too. *The strength requirement for a structure with cracks*, may be written as,

$$\sigma_{eq} \le \sigma_{al}(a;c), \qquad (7)$$

where $\sigma_{al}=(a;c)$ is the allowable stress of the cracked specimen,

$$\sigma_{al}(a;c) = \sigma_{cr}(a;c)/c_{\sigma} .$$

$\sigma_{cr}=(a;c)$ results from the first eq. (2).

b. The strength calculation with *Fracture mechanics concepts* is based on the condition,

$$K_{eq} \le K_{al}, \qquad (8)$$

where K_{eq} is the equivalent stress intensity factor and K_{al} is the allowable value of the stress intensity factor.

c. Another way to solve the strength problem on the basis of Fracture mechanics concepts but using the *Principle of critical energy* consists in the following condition [23, 24],

$$\sum_i \left(\frac{K_i}{K_{i,al}}\right)^{\alpha+1} \le 1, \qquad (9)$$

where i =I, II, III refer to the three fracture modes: I – opening mode; II – sliding mode; III – tearing mode.

d. The *strength calculation method based on the Principle* of *critical energy* shows that the loading state must meet the condition [17],

$$P_T^* \le P_{al}, \qquad (10)$$

where the total participation due to stresses superposition with respect to the allowable state is,

$$P_T^* = \sum_i \left(\frac{\sigma}{\sigma_{al}}\right)_i^{\alpha+1} \cdot \delta_{\sigma_i} + \sum_j \left(\frac{\tau}{\tau_{al}}\right)_j^{\alpha_i+1} \cdot \delta_{\tau_j}, \qquad (11)$$

where $\sigma_{al}= \sigma_{cr} / c_{\sigma}$ and $\tau_{al}= \tau_{cr} / c_{\tau}$ are the allowable stresses of uncracked specimens, while $c_{\sigma}>1$; $c_{\tau}>1$ are the safety coefficients with respect to the normal and shear stress, respectively.

The allowable participation, P_{al}, has the expression,

$$P_{al} = 1 - D_T^* - P_{res}^*, \qquad (12)$$

where $D^*_T(t)$ is the total damage with respect to the allowable state and P^*_{res} is the specific energy participation corresponding to the residual stress.

Strength of cracked structures.

In practice piping systems, as well as pressure equipment with nozzles, are always subjected to combined pressure and system loadings (pressure, bending moment, torsional moment, forces…), thus the studies need to be carried out of combined loading; it needs often to consider the superposition of the deteriorations due to cracks.

When under a group of loads such as L_i (i=1;2;3...n), *the total participation of specific energies* introduced into the structure material is written as [9 - 11],

$$P_T = \sum_i \left(\frac{L_i}{L_{i,cr}}\right)^{\alpha+1} \cdot \delta_L, \qquad (13)$$

where $L_{i,cr}$ is the critical value of the generalized load L_i, while $\delta_L = 1$ if L_i acts in the direction of the process and $\delta_L = -1$, if it opposes the evolution of the process.

The critical state is reached when,

$$P_T = P_{cr}(t), \qquad (14)$$

where the critical participation

$$P_{cr}(t) = P_{cr}(0) - D_T(t) - P_{res}. \qquad (15)$$

In eq. (15),

$$D_T(t) = \sum_k D_k(t),$$

is the total deterioration versus the critical state, time dependent, a sum of the partial deteriorations, $D_k(t)$, due to different loads/actions (corrosion, aging, erosion, crack, creep, fatigue, hydrogen, neutrons etc…).

$P_{cr}(0)$ is the value of $P_{cr}(t)$ at t=0; it takes values between $P_{cr,min}(0)>0$ and $P_{cr,max}(0) \le 0$, depending on the scatter of the material mechanical characteristics involved in the loading process. If the mechanical characteristics are deterministic values, than $P_{cr}(0)=1$.

The residual stress influence versus the critical state is introduced through the participation of residual stress (σ_{res}) specific energy, P_{res}, a dimensionless variable [9;10].

For crackless structures $D_T(t)=0$ and no residual stresses $P_{cr}(t)=P_{cr}(0)$. Consequently, the group of static loads becomes critical if,

$$\sum_i \left(\frac{L_i}{L_{i,cr}}\right)^{\alpha+1} \cdot \delta_L = P_{cr}(0). \qquad (16)$$

If a tubular structure is simultaneous loaded with internal pressure and bending moment, eq. (16) becomes,

$$\left(\frac{p}{p_{cr}}\right)^{\alpha+1} + \left(\frac{M_b}{M_{b,cr}}\right)^{\alpha+1} \cdot \delta_M = P_{cr}(0), \qquad (17)$$

where $\delta_M=1$ in the section where M_b causes elongation and $\delta_M=-1$ in the section where M_b produces compression.

For linear-elastic loading ($\sigma_{max} \le \sigma_y$, $\alpha=1 / k=1$. If one adds to the above $P_{cr}(0)=1$ corresponding to the use of the deterministic values of the mechanical characteristics, then the relationship (17) becomes,

$$\left(\frac{p}{p_{cr}}\right)^2 + \left(\frac{M_b}{M_{b,cr}}\right)^2 = 1. \qquad (18)$$

This relationship was based on experimental data obtained with tube specimens made from carbon steel (St 20) and austenitic steel (12X18H10T) [25].

The eq. (18) has been proposed for calculating the resistance of pipes featuring $\beta=R_2/R_1 \le 3$, simultaneously loaded with internal pressure *p* and bending moment, M_b [26] with following eqs. for critical parameters [25],

$$p_{cr} = \frac{2}{\sqrt{3}} \cdot \sigma_{0.2} \cdot \ln\beta \text{ and } M_{b,cr} = 2 \cdot \sigma_{0.2} \cdot S_s, \qquad (19)$$

which is the yield pressure and the yield bending moment, respectively. Stress $\sigma_{0.2} = \sigma_y$ is the yield stress corresponding to a residual strain of 0.2%; S_s is the static moment of the area section.

Strength of cracked tubular junction

In many practical cases the tubular joints welds are susceptible to crack formation (fig. 3).

Consequently, the strength of the nozzle-vessel junction affected by the cracks often experience mixed mode loading (fig. 3).

The state of a cracked tubular joint subjected to internal pressure, *p*, in-plane bending moment, $M_{b,i}$ out-of-plain bending moment, $M_{b,o}$ torsion moment, M_t and axial elongational force, *F*, becomes critical if eq. (16) is fulfilled, namely,

$$\left(\frac{p}{p_{cr}}\right)^{\alpha+1}+\left(\frac{F}{F_{cr}}\right)^{\alpha+1}\cdot\delta_F+\left(\frac{M_{b,i}}{(M_{b,i})_{cr}}\right)^{\alpha+1}\cdot\delta_{M_i}+\left(\frac{M_{b,0}}{(M_{b,0})_{cr}}\right)^{\alpha+1}\cdot\delta_{M,0}+\left(\frac{M_t}{M_{t,cr}}\right)^{\alpha_t+1}=P_{cr}(t) \qquad (20)$$

Ignoring the deterioration one obtains the overestimation of the resistance of the branch junction, which is not the case of eq. (20).

In the case of internal pressure and in-plane bending moment loading, neglecting the residual stress influence, and limiting the maximum stress to the value of yield stress (when k =1 and α=1/k=1), eq (20) becomes,

$$\left(\frac{p}{p_{cr}}\right)^{2}+\left(\frac{M_{b,i}}{(M_{b,i})_{cr}}\right)^{2}=P_{cr}(t). \qquad (21)$$

with $P_{cr}(t)=1$ one obtains the empirical eq. (18).

A branch junction (fig. 3) which had a branch/run pipe mean diameter ratio r_m / R_m = 0.5 a thickness ratio t/T=1 and diameter/ thickness ratio $2R_m$ / T = 20 were used. Through-wall cracks were considered with an angular extension 2β [27, 28]. The critical loads assumed the limit loads, corresponding to yield stress, namely p_{cr} = p_L and $M_{b,cr}=M_{b,L}$. For uncracked model ($D_T(t)=0$) the interaction diagram is a quarter of a circle ($\sqrt{P_{cr}(t)}=1$ in fig. 4).

If 2β>0, then $D_T(t)>0$ and eq. (21) describes circles with the radius $\sqrt{P_{cr}(t)}<1$ (fig. 6).

For example for 2b=140° results a quarter of circle with the radius $\sqrt{P_{cr}(t)}=0.8$ (fig. 7).

Similar results were obtained for $2R_m$/T=10 and 30 and for $r_m/R_m=0.95$ [29].

The paper [30] describes the effect of cracks on the limit loads of branch junctions under combined pressure and bending to the branch pipe and, separate, to the run pipe. Two branch components were considered, namely (fig. 3):

- large bore, with R_m=244.5mm; T=31.8mm; r_m=154mm; t=15.9mm
- medium bore, with R_m=222.5 mm; T=20mm; r_m=59.5 mm; t = 8 mm.

For un-cracked branch junction under combined pressure and bending to the branch piping the interaction curve (p/p_{cr} versus M_b / $M_{b,cr}$) is circular and is given by eq. (18).

For through-wall crack (a=T), with the crack located on the crotch, in the lower weld toe (fig. 4), with the relative length on the crotch φ/π ≤ 0.5 the circular interaction is described by eq. (21) where $D_T(t)=D(a;\varphi)$ is the deterioration due to crack. The circles are of radius $\sqrt{P_{cr}(t)}=\sqrt{1-D(a;\alpha)}$, in the both cases: large bore and medium bore [13]. One can see that the points obtained in [30] fitters with eq. (22). Because $\sqrt{P_{cr}(t)}$ for medium bore,

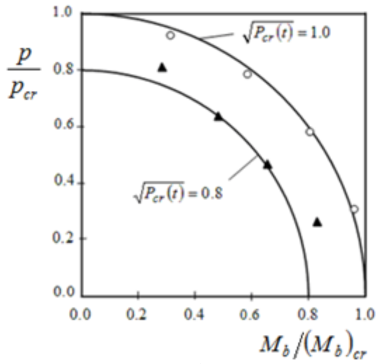

Fig. 4. Interaction diagram for pressure and bending moment loading for $2R_m$/T=20; r_m/R_m=0.5; t=T through wall crack: o uncracked; ▲ 2β=140° [27, 28]; curves drawn with eq. (21)

for the same φ/π, is higher then for large bore, one may conclude: the medium bore junction has a higher strength then the large bore junction.

Strains and stresses by a tubular junction

We devised a tubular junction (fig. 5) out of a pipeline (*1*) and two welded identical diametrically opposed nozzles (*2* and *3*) welded. On the junction components there were glued 10 resistive electric transducers (TER).

The resistive electric transducers, Hottinger brand, type 1 - XY11-3 / 120 with a 3 mm base measure deformations along two perpendicular directions (*z* – meridional or axial – and θ circumferential). The TERs were coupled to the channels of a portable data acquisition system *MGC – plus* equipped with CATMAN software.

At the bottom of branch *3* there was generated a crack, *f*, along the pipe run *1* tangent to the weld *6*.

The transducers next to crackless branch *2* are written as R_3 (on nozzle *2*), R_2, R_9 and R_{10} (on pipeline *1*). Transducers close to crack *f* at the base of the weld between nozzle *3* and pipeline *1* are written as R_6 (on nozzle *3*), R_5, R_7 and R_8 on pipeline *1*.

Transducers R_1 and R_4 are located away from the crack and the pipe – nozzles junction, consequently away from their influence.

On the basis of meridional strain ε_z, and circumferential ones $\varepsilon_è$, respectively, experimentally determined, one calculated the meridional σ_z and circumferential $\sigma_è$, stresses using Hooke's relations for stress state,

$$\left.\begin{aligned}\sigma_z&=\frac{E}{1-\nu^2}(\varepsilon_z+\nu\cdot\varepsilon_\theta);\\ \sigma_\theta&=\frac{E}{1-\nu^2}(\varepsilon_\theta+\nu\cdot\varepsilon_z).\end{aligned}\right\} \qquad (22)$$

where E=2.1.10^5 MPa is the Young's modulus of elasticity; ν=0.3 - Poisson's ratio.

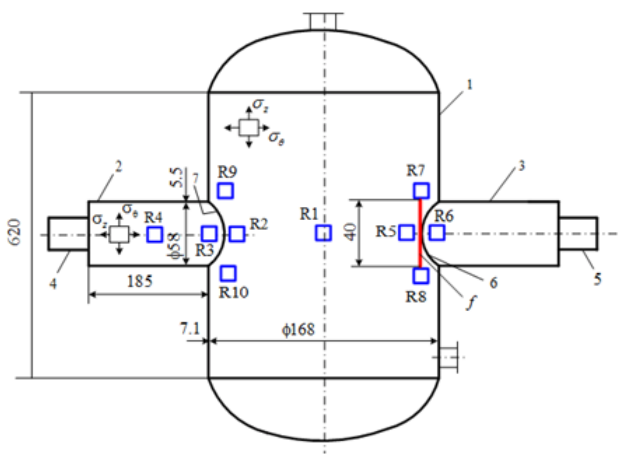

Fig. 5. Experimental steel model: 1 – run pipe; 2, 3 - nozzles; 4; 5 - plugs; 6; 7 – weld; f - crack.

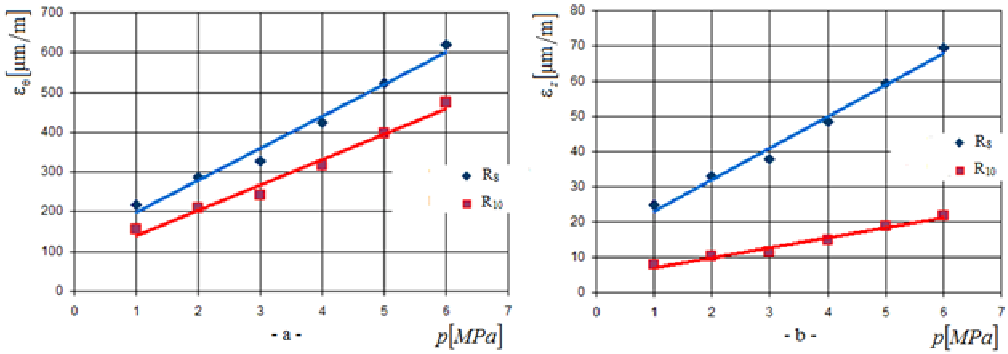

Fig. 6. Strain on circumferential (*a*) and meridional (*b*) directions dependent on internal pressure for measuring point R_8 found at the crack tip and the homologous, uncracked point R_{10}.

The experimental model was pressurized to maximum p=6MPa. It has been found that strains increases linearly with increasing pressure (fig. 6), which indicates that the loading lies in the elastic range both at *point* R_{10} next to uncracked nozzle *2*, as well as at the homologous point R_8 at the crack tip located next to nozzle *3*.

Consequently, one may apply relations (22).

Stresses σ_z and σ_θ calculated with equations (22) are also linear dependent on the pressure.

For example, stress σ_z and σ_θ at the R_1 *point* of measurement varies linearly with the strains and through their agency varies linearly with pressure *p*.

TER R_1 and R_4 make it possible to calculate the stresses (written (σ_z(p) and σ_θ(p)), on both the pipeline and the nozzle), unaffected by the weld and the crack.

TER R_7 and R_8 on the pipe - at a given pressure - may be considered to reveal only the influence of crack *f*. The corresponding stresses are written σ_z(p;a;c) and σ_θ(p;a;c).

TER R_2 on the pipe and R_3 on nozzle *2* allow the calculation of stresses (written, $\sigma_z(p;\alpha_\kappa;\sigma_{res,e})$ and $\sigma_\theta(p;\alpha_\kappa;\sigma_{res,e})$) that - at a given pressure - are influenced by the stress concentration (α_k) represented by the connection between bodies *1* and *2* and the residual stresses caused by welding (σ_{res}).

TER R_5 on the run pipe *1* and TER R_6 on nozzle *3* allow the calculation of the stresses (written $\sigma_z(p;\alpha_\kappa;\sigma_{res,e};a;c)$ and $\sigma_\theta(p;\alpha_\kappa;\sigma_{res,e};a;c)$ which - at a given pressure - are influenced by stress concentration (α_k), residual stresses (σ_{res}) and crack *f* (depth *a* and length .

Table 1 lists the stress values calculated from relations (23); the loading state is linear - elastic, since the maximum stress (161.61 MPa) is less than the yield strength of the junction material (σ_y=245MPa).

Stress concentrations and deterioration at measuring points

The comparison of stress measurement results allows the calculation of stress concentrations:

– *stress concentration coefficients* determined by the pipe–nozzle junction,

$$\alpha_{k,z} = \frac{\sigma(p;\alpha_k)}{\sigma_z(p)} \quad \text{and} \quad \alpha_{k,\theta} = \frac{\sigma_\theta(p;\alpha_k)}{\sigma_\theta(p)}; \qquad (23)$$

–*stress concentration coefficients determined only by the crack,*

$$\beta_{k,z} = \frac{\sigma_z(p;a;c)}{\sigma_z(p)} \quad \text{and} \quad \beta_{k,\theta} = \frac{\sigma_\theta(p;a;c)}{\sigma_\theta(p)}; \qquad (24)$$

–*stress concentration coefficients when considering experimental residual stresses, $\sigma_{res,e}$,*

· instead of $\alpha_{k,z}$ and $\alpha_{k,\theta}$ one calculates,

$$\alpha'_{k,z} = \frac{\sigma(p;\alpha_k;\sigma_{res,e})}{\sigma_z(p)} \quad \text{and} \quad \alpha'_{k,\theta} = \frac{\sigma_\theta(p;\alpha_k;\sigma_{res,e})}{\sigma_\theta(p)}; \qquad (25)$$

· instead of $\beta_{k,z}$ and $\alpha_{k,\theta}$ one calculates

$$\beta'_{k,z} = \frac{\sigma_z(p;a;c;\sigma_{res,e})}{\sigma_z(p)} \quad \text{and} \quad \beta'_{k,z} = \frac{\sigma_\theta(p;a;c;\sigma_{res,e})}{\sigma_\theta(p)}, \qquad (26)$$

–*stress concentration coefficients* caused by pipe-nozzle junction (α_k), experimental residual stress ($\sigma_{res,e}$) and crack, (a,c)

$$\gamma_{k,z} = \frac{\sigma_z(p;\alpha_k;\sigma_{res,e};a;c)}{\sigma_z(p)} \quad \text{and} \quad \gamma_{k,z} = \frac{\sigma_\theta(p;\alpha_k;\sigma_{res,e};a;c)}{\sigma_\theta(p)}. \qquad (27)$$

Table 1

MERIDIONAL σ_z AND CIRCUMFERENTIAL σ_θ STRESS VALUES AT THE *POINTS* WHERE TERs ARE LOCATED WHEN THE TUBULAR JUNCTION UNDERGOES STRESSES AT INTERNAL PRESSURE p = 6MPa

	R1	R2	R3	R4	R5	R6	R7	R8	R10
$\sigma_z(p)$	25.26			11.81					
$\sigma_\theta(p)$	68.86			36.53					
$\sigma_z(p;\alpha_k)$									38.1
$\sigma_\theta(p;\alpha_k)$									111.17
$\sigma_z(p;a;c)$							60.4	59.1	
$\sigma_\theta(p;a;c)$							161.6	148.1	
$\sigma_z(p;\alpha_k;\sigma_{res})$		111.44	63.34						
$\sigma_\theta(p;\alpha_k;\sigma_{res})$		136.51	-23.01						
$\sigma_z(p;\alpha_k;\sigma_{res};a;c)$					109.92	41.29			
$\sigma_\theta(p;\alpha_k;\sigma_{res};a;c)$					104.84	-31.87			

For example TER R_5 °i R_6 are at the *points* where all the three influences are at play: stress concentration due to the branch-pipe junction, residual stresses and the crack.

As in the case under analysis the maximum stresses were lower than the yield limit, *the loading lies in the linear-elastic range*. As a result, at any point of the structure the stresses can be summed up algebraically. Consequently:

- the stress determined by the stress concentration (branch–pipeline joint) under the action of pressure p,

$$\sigma(p;\alpha_k)=\sigma(p)+\Delta\sigma(p;\alpha_k), \tag{28}$$

where σ(p) is the stress produced by pressure *p*, while $\Delta\sigma(p;\alpha_k)$ is the *local increase* (under the action of pressure *p*) generated by the local stress concentration (α_k) represented by the joint of the two tubular elements (*1* and *2*, or *3*);

- the stress at the crack tip, away from the stress concentration and unaffected by residual stresses,

$$\sigma(p;a;c)=\sigma(p)+\Delta\sigma(p;a;c), \tag{29}$$

where Δσ(p;a;c) is the *local stress increase* (under pressure *p*) imposed only by the presence of the crack $(a;c)$;

- stress in crack area, away from the stress concentration, affected by the residual stress,

$$\sigma(p;a;c;\sigma_{res,e})=\sigma(p)+(\Delta\sigma(p;a;c)+\sigma_{res}), \tag{30}$$

- stress induced by the internal pressure in the vicinity of the stress concentration, affected by the residual stress, $\sigma_{res,e}$,

$$\begin{aligned}\sigma(p;\alpha_k;\sigma_{res,e})&=\sigma(p;\alpha_k)+\sigma_{res}=\\&=\sigma(p)+(\Delta\sigma(p;\alpha_k)+\sigma_{res})\end{aligned} \tag{31}$$

where $\sigma_{res,e}$ is the local residual stress;

- stress under the action of inner pressure, influenced by the stress concentration (nozzle - pipeline connection), the presence of the crack *(a, c)* and the residual stress ($\sigma_{res,e}$),

$$\begin{aligned}\sigma(p;\alpha_k;a;c;\sigma_{res,e})&=\sigma(p;\alpha_k;\sigma_{res,e})+\Delta\sigma(p;a;c)=\\&=\sigma(p)+(\Delta\sigma(p;\alpha_k)+\Delta\sigma(p;a;c)+\sigma_{res})\end{aligned} \tag{32}$$

From previous relationships one can *extract* the local variations in the stresses caused by:

- stress concentration,

$$\Delta\sigma(p;\alpha_k)=\sigma(p;\alpha_k)-\sigma(p); \tag{33}$$

- crack,

$$\Delta\sigma(p;a;c)=\sigma(p;a;c)-\sigma(p); \tag{34}$$

- calculated residual stresses,

$$\sigma_{res}=\sigma(p;\alpha_k;\sigma_{res,e})-\sigma(p;\alpha_k) \tag{35}$$

or

$$\sigma_{res}=\sigma(p;\alpha_k;a;c;\sigma_{res,e})-[\sigma(p)+\Delta\sigma(p;\alpha_k)+\Delta\sigma(p;a;c)]. \tag{36}$$

The values of the stress concentrations listed in table 1 are further calculated with relations (23-26).

The equivalent stress σ_{eq}, at each point is calculated by referring to Tresca's theory,

$$\sigma_{ech}=\sigma_{max}-\sigma_{min}\approx\sigma_{max}, \tag{37}$$

where one considered $\sigma_{min}\approx 0$ (radial tension).

- With TER R_{10} on pipeline *1*, α_{kz}=1.508 and $\alpha_{k\theta}$ = 1.614.
- With TER R_3, on nozzle *2*, α'_{kz}=5.363 and $\alpha'_{k\theta}$=(-)0.630, which shows that on the joining contour the circumferential stresses induced by the contour loads are of the compressive ones;
- With R_7 °i R_8 along pipeline *1*, at the tip of the crack:

$$\beta_{k,z}=\begin{cases}2.392, \text{ for } R_7;\\2.340, \text{ for } R_8;\end{cases}\quad \beta_{k,\theta}=\begin{cases}2.347, \text{ for } R_7;\\2.150, \text{ for } R_8.\end{cases}$$

- With TER R_2 on pipeline 1, wherein residual stresses come into play, besides the stress concentration one calculates

$$\alpha'_{k,z}=4.53 \quad\text{and}\quad \alpha'_{k,\theta}=1.982 .$$

With TER R_5 on pipeline 1 and R_6 on nozzle *3*, there mixes in the influence of the crack adjoining the stress concentration and the residual stresses. One obtains the following stress concentration values:

$$\gamma_{k,z}=\begin{cases}4.352, \text{ for } R_5, \text{on pipeline } 1;\\3.496, \text{ for } R_6, \text{on nozzle } 3.\end{cases}$$

$$\gamma_{k,\theta}=\begin{cases}1.523, \text{ for } R_5, \text{on pipeline } 1;\\(-)0.8724, \text{ for } R_6, \text{on nozzle } 3.\end{cases}$$

One can notice that with TER R_6 on nozzle *3* the circumferential stress is negative, for the same reasons as those for R_3 on nozzle *2*. This finding was proved by the stress concentration coefficients by bracketing (-) the minus sign.

The highest values of the stress concentrations corresponds to the meridional direction on nozzle *2* (R_3), on nozzle *3* (R_6) and on pipeline *1* (R_2 °i R_5).

Conclusions

The paper refers to the strength calculation of tubular junctions. In this context, one has undertaken the analysis of stress concentrations around the pipe - branch junction.

One analyzes first the state of the art in this field and one rationalizes the relations recently proposed for the calculation of the critical stresses and loads in structures with cracks. Based on these data one has presented the calculation methods for structure strength, namely: - classical calculation based on strength theory; - calculations based on fracture mechanics concepts; - calculation based on the relationship obtained by using the *principle of critical energy*.

One has found that the calculation method based on the principle of critical energy is the only calculation method likely to account for: - the nonlinear behavior of the material; - the damage caused by the cracks; - the influence of residual stresses; - the stochastic distribution of the values of mechanical characteristics of the material structure.

One has put forth a calculation relationship for the strength of tubular junctions based on the principle of critical energy. The relationship obtained was verified with experimental data from the literature for tubular structures with cracks and tubular junctions with cracks under multiple simultaneous loads (internal pressure, axial force, bending moment, torque). The theoretical results based on the principle of critical energy is in good agreement with the experimental results reported in literature.

On a tubular junction designed with two diametrically opposed nozzles, including one with a crack in the weld base, there were located strain gauges. This structure was gradually subjected to a series of inner pressure values between 0 and 6 MPa. Stress concentration coefficients have been defined by considering the influence of the internal pressure, fracture and residual stresses.

Based on measurements there were determined the stresses in the meridional and the circumferential directions. By using the defining relations of stress concentrations, these values were calculated based on experimental data. It has been found that the highest values of stress concentrations pertain to the meridional direction both on the pipe as well as on the two nozzles.

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